Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

305. Proposed by **C. N. SCHMALL**, New York City.

Prove $\int_{\beta}^{x}\frac{dx}{\sqrt{[(\alpha-x)(x-\beta)]}}=2\cos^{-1}\sqrt{\frac{\alpha-x}{\alpha-\beta}}$. [Edwards' *Integral Calculus for Beginners*, p. 84, ex. 4.] Does this result hold when the upper limit is changed from x to α?

Solution by **A. M. HARDING**, Associate Professor of Mathematics. University of Arkansas.

Put $x=\beta\cos^2\theta+\alpha\sin^2\theta$, *i. e.*, $\theta=\cos^{-1}\sqrt{\frac{\alpha-x}{\alpha-\beta}}$; then $\alpha-x=(\alpha-\beta)\cos^2\theta$, $x-\beta=(\alpha-\beta)\sin^2\theta$, $dx=(\alpha-\beta)\sin 2\theta\, d\theta$.

Hence, $$\int_{\beta}^{x}\frac{dx}{\sqrt{[(\alpha-x)(x-\beta)]}}=\int_{0}^{\theta}\frac{(\alpha-\beta)\sin 2\theta\, d\theta}{(\alpha-\beta)\sin\theta\cos\theta}$$

$$=2\int_{0}^{\theta}d\theta=2\theta=2\cos^{-1}\sqrt{\frac{\alpha-x}{\alpha-\beta}}.$$

When the upper limit is α, we have

$$\int_{\beta}^{\alpha}\frac{dx}{\sqrt{[(\alpha-x)(x-\beta)]}}=\operatorname*{Lim.}_{\varepsilon\doteq0}\int_{\beta}^{\alpha-\varepsilon}\frac{dx}{\sqrt{[(\alpha-x)(x-\beta)]}}$$

$$=\operatorname*{Lim.}_{\varepsilon\doteq0}2\cos^{-1}\sqrt{\frac{\alpha-(\alpha-\varepsilon)}{\alpha-\beta}}=2.\tfrac{1}{2}\pi=\pi.$$

Also solved in various ways by C. E. Rust, A. H. Holmes, V. M. Spunar, and the Proposer.

MECHANICS.

253 (Incorrectly numbered 354). Proposed by **W. J. GREENSTREET, M. A.**, Editor of The Mathematical Gazette, Stroud, England.

On a smooth horizontal plane are touching each other two balls, equal, uniform, inelastic, spherical, radius x, mass M. A third uniform, smooth, elastic ball, radius y, mass m, is placed with its centre vertically above the point of contact of those balls. Its radius may vary as long as its mass remains m. When is the velocity of each inelastic ball after impact a maximum?

No solution of this problem has been received.

354 (Incorrectly numbered 355). Proposed by the late **G. B. M. ZERR, Ph. D.**

Find the current given by a battery $\frac{1}{2}n(n+1)$ cells arranged in n rows as follows: First row, 1 cell; second row, 2 cells; third row, 3 cells; nth row, n cells. The electromotive force of each cell is E and its resistance r, while the resistances of the wires joining the positive and negative ends of the rows to the poles are as follows: First row, nr; second row, $(n-1)r$; third row, $(n-2)r$, ..., $(n-2)$nd row, $3r$; $(n-1)$st row, $2r$; nth row, r. The resistance of the two wires joining the poles to complete the circuit is r for each.

Solution by V. M. SPUNAR, C. E., Chicago, Ill.

If the cells in the mth row are connected in series, the current in that row is, by Ohm's law,

$$C_m = \frac{mE}{mr+R_m},$$

where R_m is the external resistance$=2r(n-m+1)$, by the conditions of the problem. Hence, the total current is

$$C = \sum_{m=1}^{m=n} \frac{mE}{mr+2r(n-m+1)+2r} = \frac{E}{r}\sum_{m=1}^{m=n}\frac{m}{2(n+2)-m}$$

$$= \frac{E}{r}\sum_{m=1}^{m=n}\left[-1+\frac{2(n+2)}{2(n+2)-m}\right] = \frac{E}{r}\left[-n+2(n+2)\sum_{m=1}^{m=n}\frac{1}{2(n+2)-m}\right]$$

$$= \frac{E}{r}\left[2(n+2)\left(\frac{1}{2n+3}+\frac{1}{2n+2}+\frac{1}{2n+1}+\dots+\frac{1}{n+4}\right)-n\right]$$

$$= \frac{E}{r}\Big[2(n+2)\left(S_{2n+3}-S_{n+3}\right)-n\Big], \text{ where } S_p = \tfrac{1}{1}+\tfrac{1}{2}+\tfrac{1}{3}+\dots+\frac{1}{p}.$$

But $S_{2n+3}-S_{n+3} = \log(2n+3)-\log(n+3)$

$$+\frac{1}{2(2n+3)}-\frac{1}{2(n+3)}-\frac{1}{12(2n+3)^2}+\frac{1}{12(n+3)^2}\dots,$$

(see MONTHLY, Vol. XVI, p. 166)

$$= \log\left(\frac{2n+3}{n+3}\right)+\tfrac{1}{2}\cdot\frac{n}{(n+3)(2n+3)}+\tfrac{1}{4}\cdot\frac{n(n+2)}{(n+3)^2\,(2n+3)^2}+\dots$$

When $n \doteq \infty$, $S_{2n+3}-S_{n+3} \doteq \log 2$.

NOTE. Contributors are asked to send in solutions of problems 255, 256, 257, 258, and 259. 255 is on page 247, Vol. XVII.

AVERAGE AND PROBABILITY.

203. Proposed by J. EDWARD SANDERS, Weather Bureau, Columbus, Ohio.

Find the average length of a hole at random through a cone.

No solution of this problem has been received.